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# BEYOND FLAT SURFACES PARAMETRIC DERIVATIONS OF HISTORICAL ISLAMIC GEOMETRIC DESIGNS

## Abstract

This paper sets out to identify a guiding methodology and define algorithms to extend the existence of Islamic geometric designs beyond flat surfaces. The paper discusses two computational approaches to deriving various non-flat geometric compositions: Euclidean Point Extrusion and Curved Surface Fitting. The paper examines historical precedents, conducts an in-depth analysis of patterns employed to generate those elements, then establishes a computational process to explore the potential of translating 2D Islamic Geometric Designs into 3D non-flat surfaces.

## Keywords

Islamic Geometric Design, Computations, Muqarnases, Domes, Parametric Design.

# BEYOND FLAT SURFACES PARAMETRIC DERIVATIONS OF HISTORICAL ISLAMIC GEOMETRIC DESIGNS

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## ABSTRACT

This paper sets out to identify a guiding methodology and define algorithms to extend the existence of Islamic geometric designs beyond flat surfaces. The paper discusses two computational approaches to deriving various non-flat geometric compositions: Euclidean Point Extrusion and Curved Surface Fitting. The paper examines historical precedents, conducts an in-depth analysis of patterns employed to generate those elements, then establishes a computational process to explore the potential of translating 2D Islamic Geometric Designs into 3D non-flat surfaces.

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## ملخص

تهدف هذه الورقة إلى تحديد منهجية إرشادية وتعريف الخوارزميات لتوسيع وجود الأنماط الهندسية الإسلامية إلى ما وراء الأسطح المستوية. تناقش الورقة نهجين حسابيين لاشتقاق مجسمات هندسية مختلفة غير مسطحة: بثق النقطة الإقليدية وتركيب الأسطح المنحنية. تدرس الورقة الامثلة التاريخية، وتجري تحليلاً متعمقاً للأنماط المستخدمة لتوليد تلك العناصر، ثم تقدم عملية حسابية لتحويل الأنماط الهندسية الثنائية الأبعاد إلى أسطح غير مسطحة ثلاثية الأبعاد.

**الكلمات المفتاحية:** التصميم الهندسي الإسلامي ، الحوسبة ، المقرنصات ، القباب ، التصميم البارامتري.

## 1. INTRODUCTION

Patterns are one of the main defining characteristics of Islamic art and architecture. Typically, Islamic patterns can be classified into floral and geometric patterns. Islamic Geometric Patterns (IGP) are commonly constructed from geometric elements that show four recognizable characteristics: symmetry, interlacing, flow, and unboundedness (Burckhardt, 2009, Abas et al., 1995). Various materials were used to produce Islamic geometric patterns, such as brick, wood, brass, and plaster. Geometry in Islamic art and architecture flourished and evolved over time (Abdullahi & Embi, 2013). Major innovations occurred between the 10<sup>th</sup> and the 16<sup>th</sup> centuries (Bonner, 2017; Abas et al., 1995). Historical evidence suggests that mathematicians and artisans collaborated regarding the perfection of geometries (Ozdural, 2000).

Most Islamic geometric designs exist on two-dimensional, flat surfaces. However, some of the later geometries were found on a non-flat surface, such as the geometries found on some Mamluks domes in Egypt or Karatay Madrasa in Konya. Those compositions show the different levels of proficiencies with some unique geometrical construction.

This paper aims to establish a taxonomy of three-dimensional Islamic geometric design and propose a computational approach to explore the design latent space in 3D.

## 2. PRECEDENTS

The recently completed Cambridge Mosque shows another example of the creative use of geometry in Islamic architecture (figure 1, left). The ceiling design of the Mosque is based on Islamic Geometric Patterns, yet the constructional components of the free-form wood ceiling morphed in certain places forming columns (Barfield, 2021). Another example that shows the creative employment of Islamic geometric patterns is the Louver Abu Dhabi which also features geometric patterns that are placed on top of the dome (figure 1, right) (Burry, 2010). Although these two projects employed Islamic geometric designs on non-flat surfaces, still, these two projects show two different approaches to deriving three-dimensional forms. Both approaches can be traced back to historical precedents.

Muqarnas, a vaulting decorative component, shows some of the earliest attempts to employ Islamic geometric design to generate a three-dimensional composition from two-dimensional design blueprints, as shown in the Topkapi and the Tashkent Scroll (figure 2) (Necipoglu, 1996). Karbandi is another possible employment of geometric designs to derive forms beyond flat surfaces, such as the ribbed dome that exists at the Great Mosque of Cordoba (figure 3). Unlike Muqarnas, the geometry is curved as if it is being projected into a round surface (Mohammadi et al., 2018).

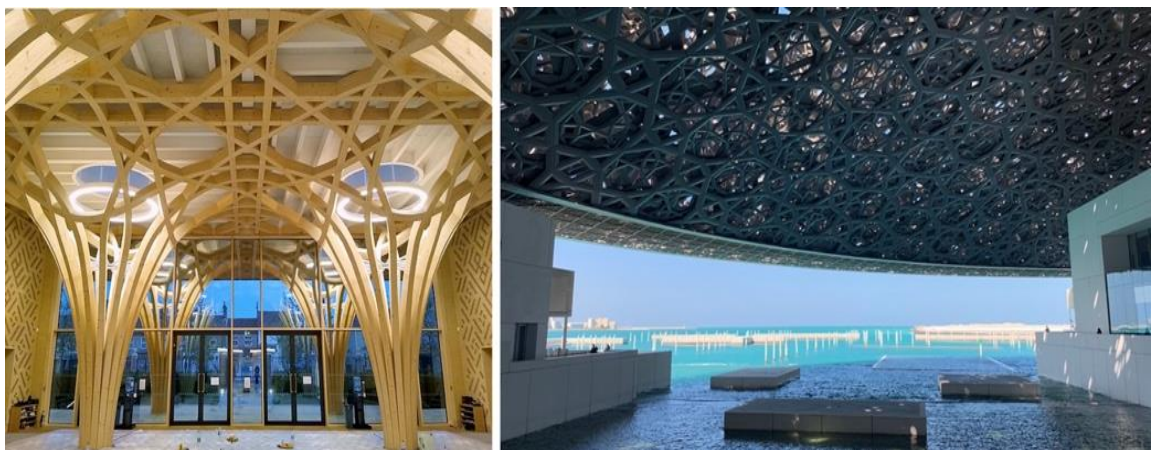


Fig. 1: Contemporary examples of IGP on non-flat surfaces. Left: Cambridge Mosque (Image credit: Wikimedia). Right: Louver Abu Dhabi (Image credit: authors).



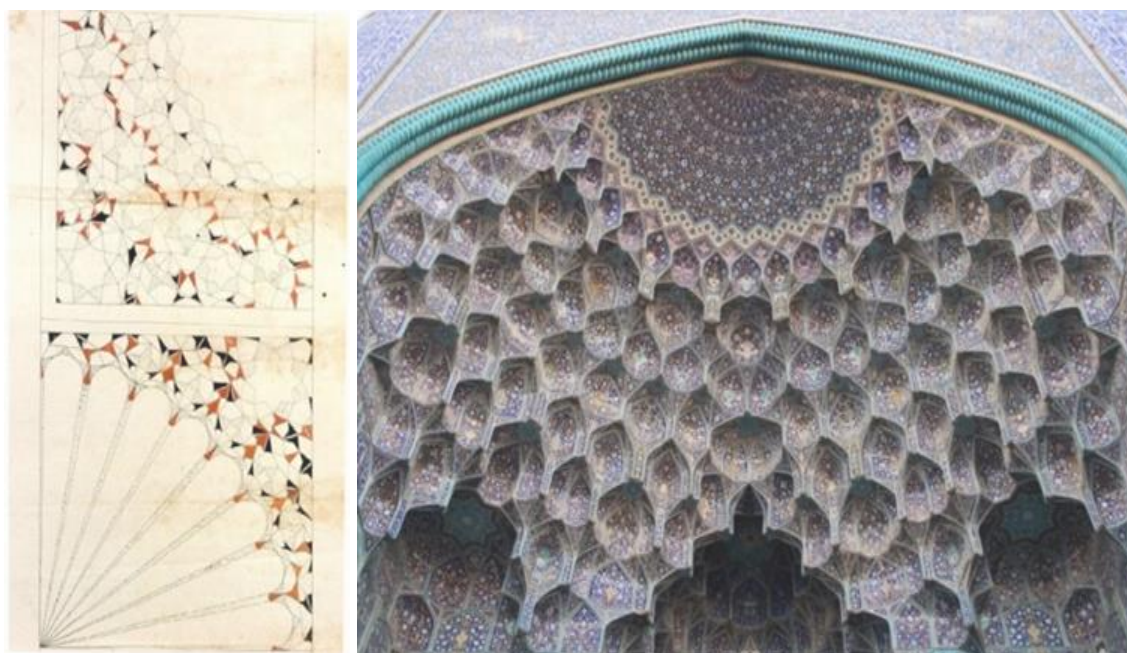


Fig.2: Left: page from the Topkapi scroll showing a 2D blueprint of a Muqarnas (Image credit: Wikimedia). Right: Muqarnas in the Iwan of the Shah Mosque in Isfahan, Iran (Image credit: Wikimedia).



Fig.3: The interior part of the dome of the Great Mosque of Cordoba (Image credit: Wikimedia).

The dome of the Karatay Madrasa (constructed in 1251 CE, Konya) features a geometric pattern that was morphed to cover the dome's interior (figure 4, left). Another remarkable example exists in Cairo at the dome of Sultan Qaytbay Funerary Complex (constructed in 1472-1474 CE) (figure 4, right). The dome is covered with several star patterns that were designed to fit the dome (Cipriani, 2005).

This quick overview of historical and contemporary precedents aims to develop an understanding of the variety of existing non-flat geometric designs by which two-dimensional geometric designs were employed to derive forms beyond flat surfaces.

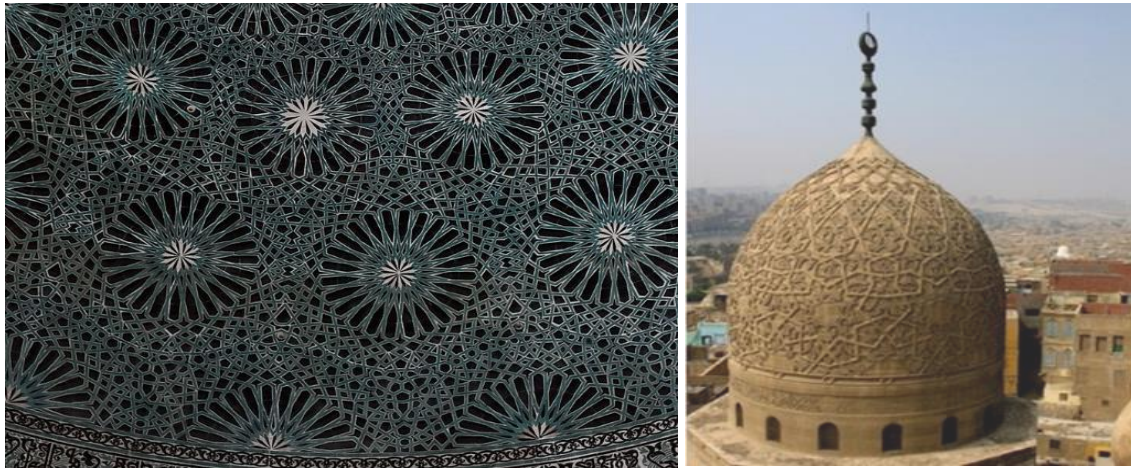


Fig.4: Historical examples of the employment of Islamic geometric patterns on non-flat surfaces. Left: the dome of the Karatay Madrasa (Image credit: Wikimedia). Right: the dome of the Sultan Qaytbay Funerary Complex (Image credit: Wikimedia).

### 3. ANATOMY OF THE ISLAMIC GEOMETRY

Generally, Islamic Geometric Designs (IGD) consist of points connecting to form line/s and/or polyline/s replicated using symmetry to fill a specified space. Typically, Islamic geometric designs show some or all of the following recognizable characteristics: symmetry, flow, unboundedness, flow, and interlacing (Abas & Salman, 1995). Islamic Geometric designs morphologically relate to each other, and it is possible to morph designs to transform from one historically existing design to another (Alani, 2017).

Three types of symmetry structures were identified in Islamic geometric designs: rosettes, periodic, and quasi-periodic structures. Figure 5 below shows an example of three points that were connected with each other using a single polyline and experimented with to create designs using the above-mentioned three structural orders.

Rosettes patterns are formed by applying rotational symmetry to a particular geometric composition around a specified axis. This type of patterns can have several axes that goes from 1 to  $\infty$  (Shubnikov 1974). Rosettes patterns have two types: cyclic and dihedral; cyclic is formed by applying rotations to the geometric compositions, while dihedral includes using reflection geometry in addition to the rotational symmetry.

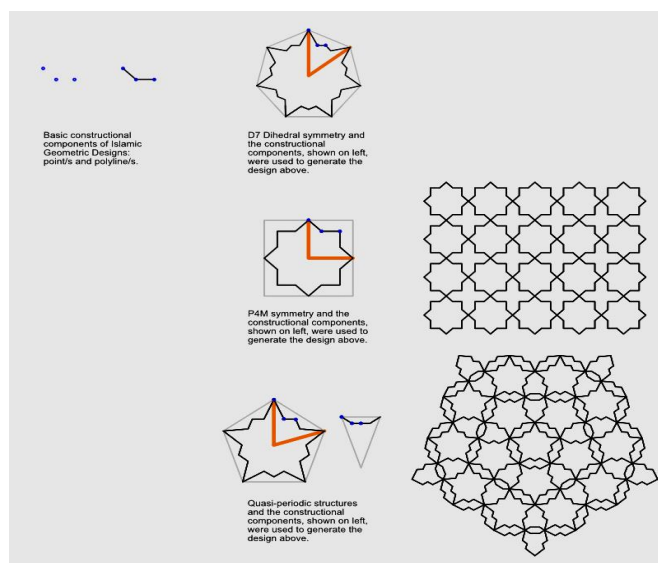


Fig. 5: shows the possible designs that can be derived from the same polylines by changing the employed symmetry (from top to bottom: rosettes, periodic, or quasi-periodic structural order).

Periodic patterns are formed through the employment of the wallpaper symmetry group to replicate geometric compositions to fill space while leaving no gaps. Islamic architecture employed the 17 possible types of the wallpaper symmetry group to create the enormous diversity of geometric designs; in fact, the seventeen possible types of wallpaper symmetry group have been identified in Alhambra palace alone (Pérez-Gómez 1987).

On the other hand, quasi-periodic patterns are also tessellated structure capable of filling the space while leaving no gaps; however, it does not employ regular translational symmetry and typically includes more than one shape (Chorbachi & Loeb 1992, Al Ajlouni 2012).

This paper codifies this definition of Islamic geometric design and explores computational processes to derive geometric forms that go beyond flat surfaces.

#### 4. MIGRATING FLAT SURFACES

The vast majority of Islamic geometric designs exist in Euclidean two-dimensional space. However, several examples of Islamic geometric designs are found on non-flat surfaces. This section discusses two methods by which Islamic geometric designs can be employed to derive three-dimensional compositions: Euclidean Point Extrusion (EPE) and Curved Surfaces Fitting (CSF).

##### 4.1 Euclidean Point Extrusion

Euclidean Point Extrusion method derives three-dimensional compositions from two-dimensional Islamic geometric designs by manipulating the points' z-coordinate with no changes to the x- and y-coordinates. The challenge in deriving the forms in this method lies in setting the value of the z-coordinate and filling any gaps that may result with appropriate forms.

Muqarnas is an exemplar case of the EPE method. Figure 6 shows an example of this method with a simple design.

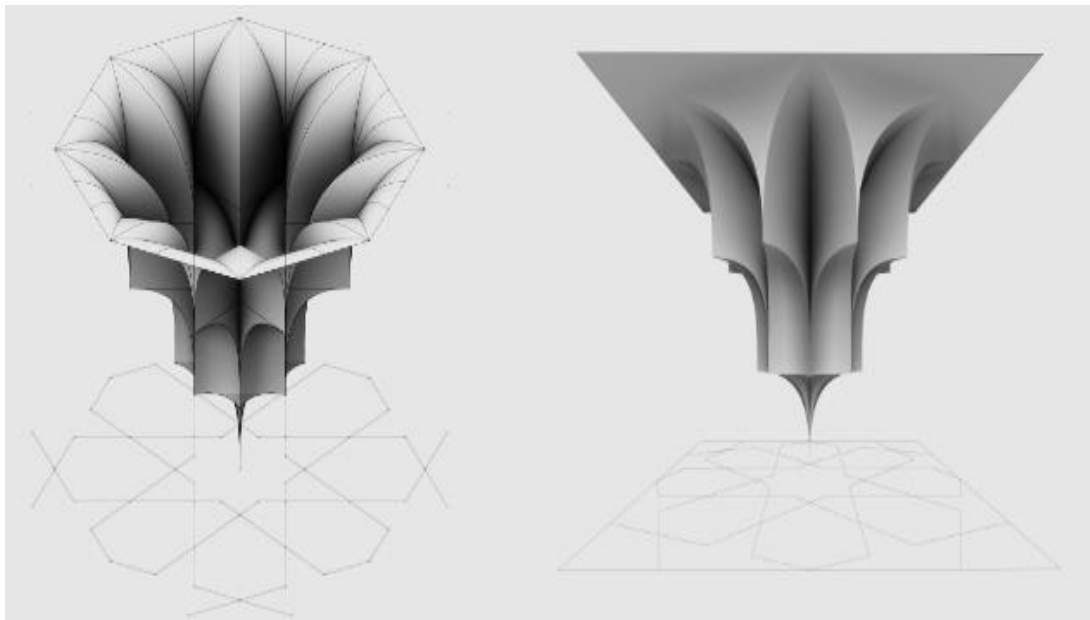


Fig.6: Square-based, Islamic geometric design was employed to generate the muqarnas-like form shown above through the manipulation of the z-coordinate of the points on different geometric patterns.

##### 4.2 Curved Surfaces Fitting:

The Curved Surfaces Fitting method aims to fit a two-dimensional Islamic geometric design into various non-flat surfaces. It can be further categorized into two sub-categories: Curve Directional Projection and Curve Mapping. Both subcategories require a Hosting Surface (HS) to project or map the design.



#### 4.2.1. Curve Directional Projection

Like the EPE method, the curve projection method also extrudes points in the z-coordinates with no changes to the x- and y-coordinates. However, the value of the z-coordinates is determined by the HS. Additionally, the curves that connect design points are generated by solving the intersection of the extruded design with the HS. Thus, the curves inherit the HS curvature while conducting no changes to the x- and y-coordinates. Note that the resulting geometry will no longer follow Euclid's Postulates as it will inherit some of the hosting geometry characteristics; for instance, the sum of the triangle angles is no longer 180 degrees) (figure 7).

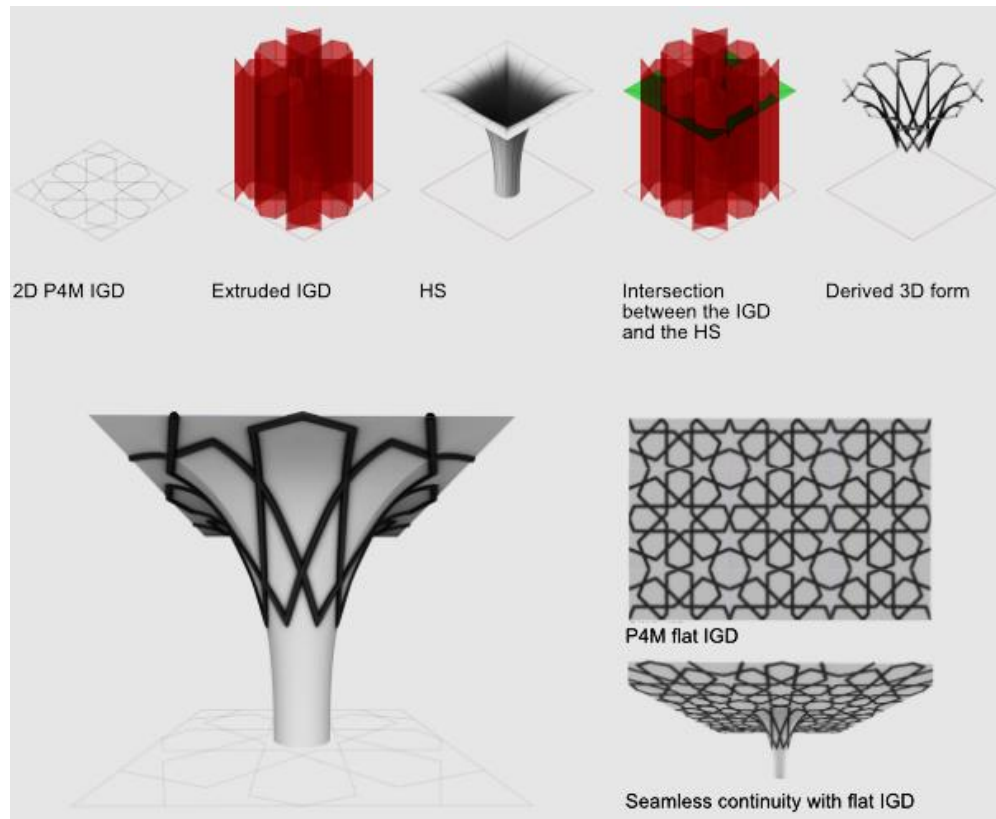


Fig.7: Left: shows example of a projected 2D Islamic Geometric Design on a curved surface. Right: tiling the derived form with the original 2D Geometric Design.

#### 4.2.2. Curve Mapping

Depending on the design problem, the curve projection method could be limiting. Consider the hemisphere shown in figure 8 (bottom left), which shows an Islamic geometric design projected into a dome-like HS. The figure clearly shows that the design is strongly distorted at the bottom boundary of the HS. This limitation is addressed by the Curves Mapping method.

The curves Mapping method would conduct significant changes to x-, y-, and z-coordinates of the design points and changes to the respective curves. That is, the points will be in a position that is relative to their original position in the two-dimensional design. Creating such forms would require extracting and flattening the HS's isoparametric curves (ISO curves). In most cases, the HS is not a developable surface. Thus, this method requires establishing a two-dimensional flat equivalent of the HS using the ISO curves. Then, the design is to be placed and fit within the flattened equivalent space and then mapped back to the HS (figure 8).



## 5. THREE-DIMENSIONAL PARAMETRIC EXPLORATION

The selection of a method to derive three-dimensional geometry depends on the design problem. For instance, if the goal is to design a dome, Curve Mapping would deliver an appropriate outcome. Figure 9 shows the computational process for deriving non-flat Islamic geometric designs. The process starts with selecting a particular Islamic geometric design to be passed to a selected migration method and deriving an output. There are two groups of parameters to work with in this flow: IGD parameters and migration method parameters.

The Islamic geometric designs parameters are the symmetry type, points count, point coordinates, and polyline intersections, all of which can explore the latent design space of a 2D Islamic geometric designs. The migration method parameters differ based on the employed method. For instance, the parameters in the Euclidean Point Extrusion are the Z-coordinates of the points and the shape of point connections. However, the parameters in the Curve Projection and Curve Mapping are primarily based on the shape of the HS. When combined, the two sets of parameters allow exploring the latent design space of Islamic geometric designs on a non-flat surface (figure 10).

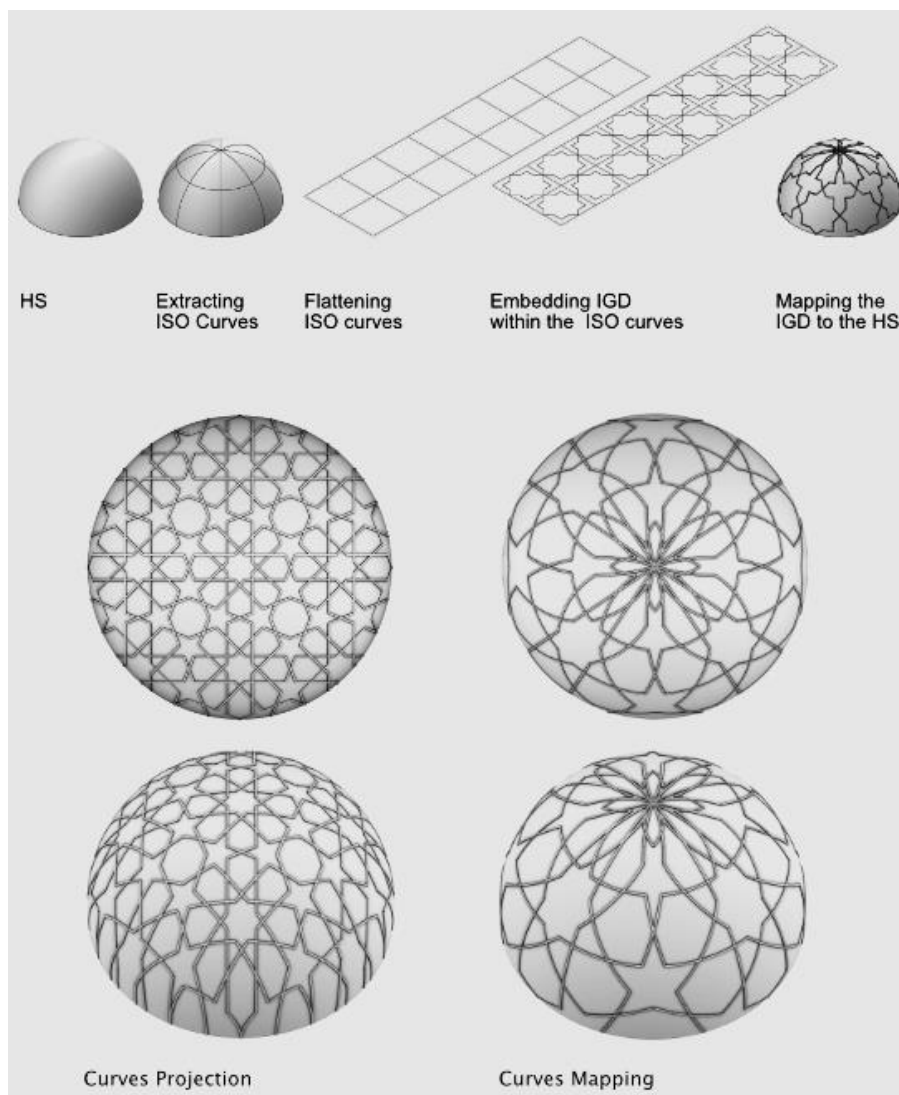


Fig.8: Curves Mapping method applied to a dome-like HS. Top: shows the process of unrolling the equivalent surface, inserting the desired design within, and mapping back the design to the hemisphere. Bottom left: perspective view illuminates the distortion occurring at the bottom edge of the HS when using the Curve Projection method. Bottom right: using Curves Mapping to fit a design into the hemisphere.

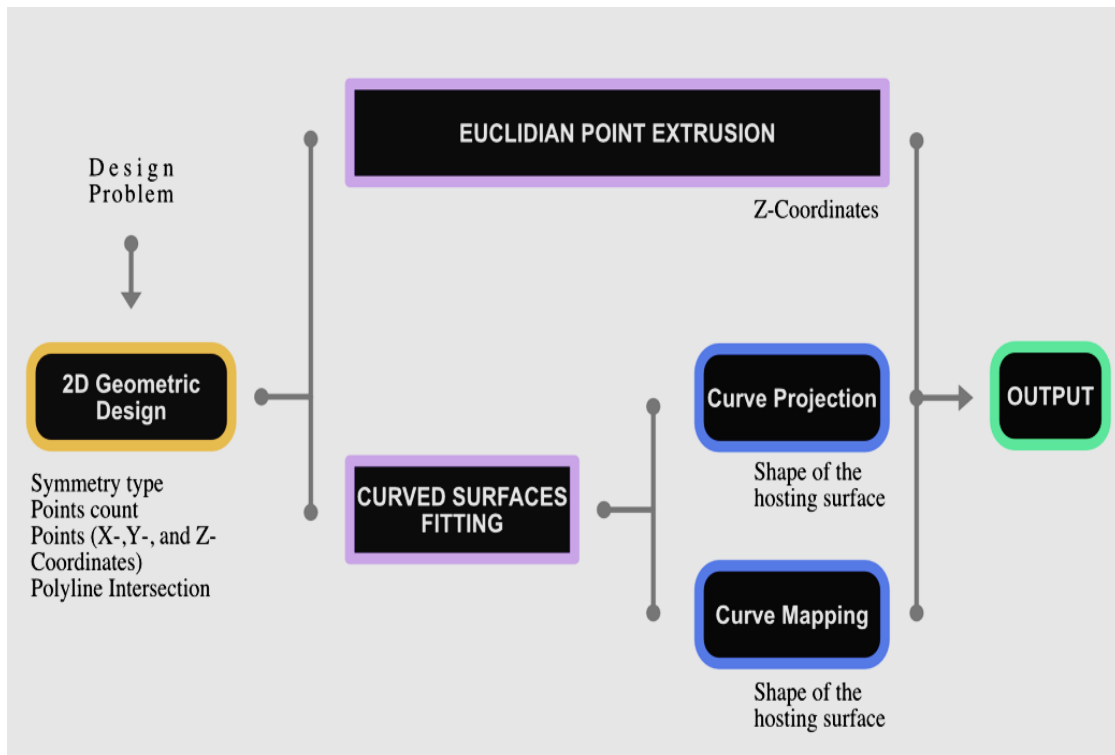


Fig.9: Computational process for exploring the latent design space of Islamic geometric design on non-flat surfaces.

## 6. DISCUSSION & CONCLUSION

In all the explored scenarios, the 2D Islamic geometric designs played a foundational role in creating the designs beyond flat surfaces. Therefore, designers' sensibility is crucial as it will affect the outcome significantly. The Euclidean Point Extrusion method allows for an additional layer for the designer to be creative in connecting design parts and filling the gaps resulting from the extrusion process. In the Curved Surfaces Fitting method, designers' creativity primarily resides at the 2D level, and the HS and all changes need to be conducted there.

Historically, Islamic architecture embedded a firm understanding of geometry that served functional and aesthetical purposes (Burckhardt, 2009). Categorizing the corpus of existing designs and identifying generative processes of such designs is necessary to understand historical designs and provide a platform to develop Islamic architecture progressively. Consequently, the aim of this study is twofold: first, the paper aims to investigate Islamic geometric designs on non-flat surfaces and categorize the different possible ways by which Islamic geometric designs can exist on non-flat surfaces; secondly, the paper seeks to unveil computational approaches by which such structures can be generated, visiting, in the process, uncharted territories to expand the search for novel forms.

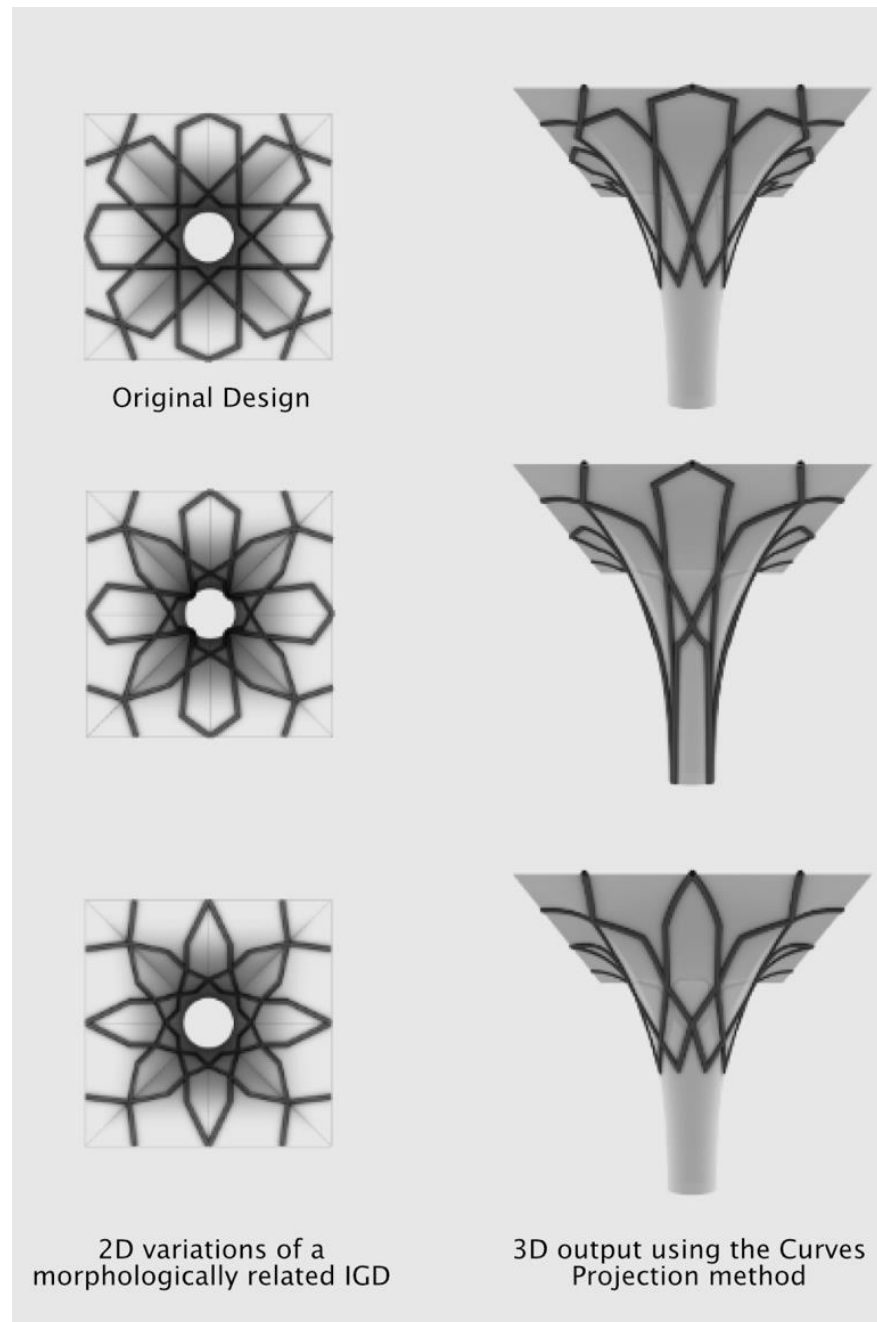


Fig.10: Design variations were created using a fourfold Islamic geometric design and the Curve Projection method.

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