CONTROL OF MR DAMPER USING ANFIS AND PID CONTROLLER FOR OPTIMUM VEHICLE RIDE COMFORT

Mohammad Faisal Yakhni  
*Eng., Faculty of Engineering, Beirut Arab University, Beirut, Lebanon*, mohammadyakhni@gmail.com

Mohamad Ali  
*Assistant Professor, Faculty of Engineering, Beirut Arab University, Beirut, Lebanon*, mohamad.ali@bau.edu.lb

Mohamed El-Gohary  
*Assistant Professor, Faculty of Engineering, Beirut Arab University, Beirut, Lebanon*, m.gohary@bau.edu.lb

Follow this and additional works at: [https://digitalcommons.bau.edu.lb/stjournal](https://digitalcommons.bau.edu.lb/stjournal)

Part of the Acoustics, Dynamics, and Controls Commons, Applied Mechanics Commons, Controls and Control Theory Commons, and the Navigation, Guidance, Control, and Dynamics Commons

Recommended Citation

DOI: [https://doi.org/10.54729/2959-331X.1001](https://doi.org/10.54729/2959-331X.1001)

This Article is brought to you for free and open access by the BAU Journals at Digital Commons @ BAU. It has been accepted for inclusion in BAU Journal - Science and Technology by an authorized editor of Digital Commons @ BAU. For more information, please contact ibthal@bau.edu.lb.
INTRODUCTION

The function of the suspension is to protect vehicle from the vibrations that it receives when traveling on a rough road surface. In general, based on the damper utilized and the actuator added to the suspension framework, this system can be sorted or classified in light on the external power input into three types, each one of them has its own advantages and disadvantages, which are: passive suspension, semi active and active suspension system (Qamar, Khan, & Qamar, 2013). The more effective suspension system is who provide greater protection and lower level of concussion moving to the car body depending on the interaction with uneven road surface (Florin, Ioan-Cozmin, & Liliana, 2013). There are various models utilized in these types of studies: quarter car model (Florin, Ioan-Cozmin, & Liliana, 2013) (Abd-Elsalam, El-Gohary, & El-Gamal, 2016) (Kalaivani, Sudhagar, & Lakshmi, 2016) (Singh, 2018) half car model (León-Vargas, Garelli, & Zapateiro, 2017) (Khodadadi & Ghadiri, 2018) and full car model (Geweda, El-Gohary, El-Nabawy, & Awad, 2017) (Yang, Liang, Zhou, & Zhao, 2019). Many researches have been made to increase and enhance ride comfort by using different active controllers. A novel Neural Network (NN) has been presented by (Kalaivani, Sudhagar, & Lakshmi, 2016) to improve the performance of the model with respect to the body acceleration under different types of road input. The response using the NN was compared to cell using conventional PID controller and a PID that was tuned using genetic algorithm and a passive suspension system. Simulations show the effectiveness of the NN compared to all these systems. A model with passenger body and seat was developed by (Singh, 2018) to capture the dynamic behavior of a car in order to improve ride comfort and safety. Two controllers, which are Adaptive Neuro Fuzzy (ANFIS) and Hybrid ANFIS PID (HANFISPID) was developed. Results demonstrate that the proposed HANFISPID control plan achieve better ride comfort and safety of travel compared to passive system and ANFIS controller. Reference (León-Vargas, Garelli, & Zapateiro, 2017) proposed a new adaptive control algorithm that combines a PID controller for suspension deflection together with a sliding mode reference conditioning outer loop that uses the vertical acceleration of the car body as complementary source of control. Results show an improvement of the ride comfort over the same PID controller without the outer conditioning loop and passive system. PID, H∞ and fuzzy logic controllers were used by (Khodadadi & Ghadiri, 2018) and developed a self-tuning PID controller based on fuzzy logic to improve the performance of the suspension system. Through the proposed controller, suspension working space (SWS) is minimized and the best comfort of the driver is achieved. Two control ways for the suspension system were presented by (Geweda, El-Gohary, El-Nabawy, & Awad, 2017). These two methods are optimization using GA to find the optimum values of spring stiffness and damping coefficient at different speeds and the other method is the control of active suspension system using Proportional Integral (PI) controller. Simulations show an important improvement in terms of sprung mass acceleration when using the presented controllers over passive system. A non-chattering sliding mode control strategy was proposed by (Yang, Liang, Zhou, & Zhao, 2019). Numerical simulation results verify that this control method is effective for the vibrations that have nonlinear characteristics of the car suspension model and achieves an improvement in ride comfort by reducing the vibration amplitude when compared to passive system.

A decent ride quality means diverse things to various individuals. Somebody who is acquainted with driving a new car will have a totally unique thought of ride quality from person who drives a multyear old pick-up. In fact, great ride quality is characterized as the capacity to limit the impacts of street irregularities to the vehicle travelers. At the point when the vehicle experiences a pothole or bump, it should transverse the impediment with little body movement as possible. The world today and mainly in Europe use the ISO 2631 as a strategy to objectively assess the ride comfort, where its quality affected by several factors which are vertical acceleration and jerks of sprung mass and passenger seat also pitch and roll acceleration of the car body and suspension travel or rattle space, while handling capability is affected by the road holding or dynamic tire load (Montazeri-Gh, Jazayeri-M, & Soleymani, 2008).

In the present study, a full passenger car model considering eight DOF is used. MR damper, that uses ANFIS to find the optimum voltage value to skip the road disturbance at least displacement, is implanted to the model to study its effect on ride comfort, especially sprung mass and seat vertical displacements. A PID controller is implanted to the model to eliminate any noise that may affect the voltage applied to the damper.
2. MODEL AND MATHEMATICAL MODEL

2.1. Full Car Model

A full car model with the driver’s seat is considered to study in this research. Fig. 1 shows an eight DOF full vehicle model, it consists of driver seat, sprung mass which refers to the part supported by springs, unsprung masses which refers to the front and rear wheels assembly. The suspension between the passenger seat and the sprung mass, also the suspension system between the sprung and unsprung masses are modeled as springs and dampers with linear characteristics. The four tires are modeled as linear spring and its damping coefficient is neglected because the damping characteristics of a tire is negligible compared to tire stiffness as indicated by (GUCLU, 2003) (Shirahatti, Prasad, Panzade, & Kulkarni, 2008) (Wang & Song, 2013). The sprung mass is considered to have three DOF which refer to pitch, roll and bounce motion. The driver seat and the four unsprung masses are modeled to have one DOF each in the vertical direction (GUCLU, 2003) (Montazeri-Gh, Jazayeri-M, & Soleymani, 2008).

2.2. Mathematical Modeling

The mathematical equations of the full car model with eight DOF and the MATLAB SIMULINK model considered by (Shirahatti, Prasad, Panzade, & Kulkarni, 2008) is used in this study. These equations were obtained using Newton’s second law of motion and free-body diagram concept. The labels and parameters adopted in this model are that have been used by (Shirahatti, Prasad, Panzade, & Kulkarni, 2008).

2.3. Magneto Rheological (MR) Damper

MR fluid can be used as one of the most important engineering application which is the construction of damper. This device is called MR linear damper, it is controllable and smart damper. These features can be achieved by applying voltage or current to the damper for creating magnetic field that changes the yield strength of the MR fluid and therefore the viscosity of the liquid. So the important asset of this damper is the controllability which can be adjusted to achieve the desired amount of dissipating energy or damping level. Obtaining the desired level of damping is achieved by applying a voltage or current to vary the magnetic induction in an orifice that separate the two chambers of the MR fluid, so the viscosity of the fluid that passes thorough this orifice changes, which affects the damping coefficient of this damper.
2.3.1. Magneto rheological fluids

There are a group of fluids which differ from other typical fluids, they have special characteristics and called MR fluids. They are non-Newtonian rheological stable with shear yield strength and are controlled by applied a magnetic field. This type of fluids is consisting of small particles that have magnetic characteristics and dispersed in a liquid, so the properties of this fluid can be changed or controlled by applying an external magnetic field. The concentration of the magnetic particles, their shape and size are parameters that determine the properties of the MR liquid in addition to the temperature, the intensity of the magnetic field and the properties of fluid carriers. Some materials that act as fluid bearers are water, mineral oil, silicon and glycerol. The diameter of the magnetic particles is between 0.5µm and 8µm (Attia & N.M. Elsodany, 2017).

2.3.2. Spencer model

Spencer proposed a model with an extension of the Bouc-Wen model, concerns the presentation of an extra spring \((K_1)\) and damper \((C_1)\) as shown in Fig. 2 (Sapiński & Filuš, 2003).

\[ F = C_0 (\dot{X} - \dot{y}) + K_0 (X - y) + K_1 (X - X_0) \]  

Or can be also expressed as:

\[ F = C_1 \dot{y} + K_1 (X - X_0) \]

The displacement \(x\) and \(y\) are respectively described by Eqs. (3-4):

\[ \dot{x} = -y|\dot{X} - \dot{y}|z|z|^{n-1} - \beta(\dot{X} - \dot{y})z^n + A(\dot{X} - \dot{y}) \]

\[ \dot{y} = \frac{1}{C_0 + C_1} [az + C_0 \dot{X} + K_0 (X - y)] \]
Where:

\[ C_0 = C_{0a} + C_{0b} U \quad \alpha = \alpha_a + \alpha_b U \]
\[ C_1 = C_{1a} + C_{1b} U \quad \dot{U} = -\mu(U - v) \]

\( C_1 \) Parameter that suits viscous damping at high speed.
\( K_1 \) Represents the stiffness at high speed.
\( \beta, \gamma, A \) Parameters representing the control of the linearity during unloading and the smoothness of transition from the pre-yield to post-yield area.
\( \alpha \) Parameter representing the stiffness of the spring for damping force component associated with the evolution variable \( z \).
\( K_0 \) Parameter representing the stiffness of the spring associated with the nominal damper due to accumulator.
\( C_0 \) Parameter representing viscous damping.
\( X_0 \) Parameter representing the initial displacement of the spring with the stiffness \( K_0 \).

The values of the above parameters of MR damper are taken as (Mahmoud El-Kafafy, 2012) and shown below:

**Eq. (5)**
\[ f_1 = C_1 \dot{y} + K_1 (Z - Z_1 - a\dot{\theta} - w\dot{\phi}) \]

**Eq. (6)**
\[ f_2 = C_1 \dot{y} + K_1 (Z - Z_2 + b\dot{\theta} - w\dot{\phi}) \]

**Eq. (7)**
\[ f_3 = C_1 \dot{y} + K_1 (Z - Z_3 - a\dot{\theta} + w\dot{\phi}) \]

**Eq. (8)**
\[ f_4 = C_1 \dot{y} + K_1 (Z - Z_4 + b\dot{\theta} + w\dot{\phi}) \]

**Eq. (9)**
\[ \dot{y} = \frac{1}{C_1 + C_0} [aJ + K_0(Z - a\dot{\theta} - W\dot{\phi} - y) + C_0(Z - a\dot{\theta} - W\dot{\phi}) - W\dot{\phi}]] \]

**Eq. (10)**
\[ J = A(Z - a\dot{\theta} - W\dot{\phi} - \dot{y}) - B\|J\|^2(Z - a\dot{\theta} - W\dot{\phi} - \dot{y}) - \gamma|Z - a\dot{\theta} - W\dot{\phi} - \dot{y}|\|J\| \]

**Eq. (11)**
\[ C_0 = C_{0a} + C_{0b} U \]

**Eq. (12)**
\[ C_1 = C_{1a} + C_{1b} U \]

**Eq. (13)**
\[ \alpha = \alpha_a + \alpha_b U \]

**Eq. (14)**
\[ \dot{U} = -\mu(U - v) \]

Where: \( U \) is the voltage applied to the damper.

The values of parameters of MR damper are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>58</td>
<td>( C_{0b} )</td>
<td>1803 ( N.s/m )</td>
<td>( \alpha_b )</td>
<td>38430 ( N/m )</td>
</tr>
<tr>
<td>( C_{1a} )</td>
<td>14649 ( N.s/m )</td>
<td>( K_0 )</td>
<td>3610 ( N/m )</td>
<td>( B )</td>
<td>2059020 ( /m^2 )</td>
</tr>
<tr>
<td>( C_{1b} )</td>
<td>34622 ( N.s/m )</td>
<td>( K_1 )</td>
<td>840 ( N/m )</td>
<td>( \gamma )</td>
<td>136320 ( /m^2 )</td>
</tr>
<tr>
<td>( C_{0a} )</td>
<td>784 ( N.s/m )</td>
<td>( \alpha_a )</td>
<td>12441 ( N/m )</td>
<td>( \mu )</td>
<td>190 ( /s )</td>
</tr>
</tbody>
</table>
3. SIMULATIONS AND CONTROL METHODS

3.1. Matlab Simulink Model

The full vehicle model is created using Matlab/Simulink (R2014b Simulink 8.4) Fig. 3 shows the flow chart from the input (road profile and speed) to the car body and finally the outputs (seat and sprung mass vertical displacements).

![Fig.3: The Simulink model flow chart](image)

3.2. Model Validation

To check the correctness of this model a comparison has done between the response of this model and that of (Shirahatti, Prasad, Panzade, & Kulkarni, 2008) in term of sprung mass displacement (Z) with respect to time when the same inputs (road profile), shown in Fig. 4, were applied to these two models.

Responses obtained are shown in Fig. 5. By comparing them, results show that the response of the Simulink model is in agreement with that of the reference. So, the eight DOF model created in this study matches (Shirahatti, Prasad, Panzade, & Kulkarni, 2008), this means that it is valid and can be used to do all needed simulations.

![Fig.4: Road profile used for model validation](image)

Reference: (Shirahatti, Prasad, Panzade, & Kulkarni, 2008)
3.3. Road Profile

A speed hump is a raised area in the roadway pavement surface extending transversely across the travel way. Speed humps are sometimes referred to as “pavement undulations” or “sleeping policemen”. Common speed hump shapes are sinusoidal, parabolic and circular. Most agencies implement speed humps with a travel length of 12 to 14 feet (3.7 to 4.3 m). Speed humps are generally used on residential local streets.

In this study a speed hump that has a half sine wave shape with 4 m wavelength. The amplitude of this hump is changed from 0.01 m to 0.2 m to study the effect of these different amplitudes on the vehicle. This hump is used as input to the car model and the vehicle is moving with variable speeds from 20 to 120 km/h, in each change to the hump amplitude, these speeds are adopted. The road hump shape is shown in Fig. 6 for 0.1 m (as example of one case of all cases used in this study) amplitude and for different speeds. The model is assumed to be constant and the hump is moving under it with variable speeds, so the hump time changes from one speed to another. In the simulation stage the time response is obtained. There is a time delay between the front and rear wheel inputs. The time delay is shown in Eq. (15):

\[
Eq. (15) \quad t = \frac{a + b}{v}
\]

Where: \((a + b)\) is the distance between the front and rear axles and \(v\) if the vehicle speed.
3.4. Adaptive-Network-based Fuzzy Inference System (ANFIS)

Modelling of a system dependent on regular numerical tools (e.g., differential equations) isn't appropriate for managing badly characterized and questionable systems. By contrast, a fuzzy inference system utilizing fuzzy if-then rules can demonstrate the qualitative parts of human information and thinking forms without utilizing exact quantitative examinations. This fuzzy modelling or fuzzy distinguishing proof, first investigated methodically by Takagi and Sugeno, has discovered various practical applications in control expectation and inference. However, there are some basic aspects of this approach which are in need of better understanding (Jang, 1993).

3.5. Proportional Integral Derivative (PID)

A PID controller is a nonexclusive control loop feedback mechanism broadly utilized in industrial control frameworks. An "error" as distinction value between a deliberate procedure variable and a desired set point is calculated by the controller. Then it attempts to limit the error by altering the procedure control inputs. The PID parameters utilized in the calculation must be tuned taking into consideration the nature of the system to achieve better performance. The PID controller calculation includes three separate parameters: the proportional, the integral and the derivative values, signified $K_p$, $K_i$, and $K_d$. The response to the current error is controlled by the estimation of the proportional parameter, while the response dependent on the sum of recent errors is dictated by the integral term value and the derivative controller anticipates the future behavior of the error. The control component utilizes the weighted sum of these three actions to adjust the process of the framework. The controller can provide control action intended to process necessities by tuning the parameters of the PID controller. The reaction of the controller can be portrayed as far as the responsiveness of the controller to an error, how much the controller overshoots the set point and the level of the system oscillations (Geweda, El-Gohary, El-Nabawy, & Awad, 2017).

3.6. Control Strategy

This section discusses the control method that has been adopted to achieve ride comfort. As mentioned above, the desired goal is achieved by minimizing, as possible, the impact of the vibrations that are transferred to the body of the vehicle and thus to the driver and passengers, resulting from the uneven road surface.

The controller that has adopted in this research to perform this purpose is the MR damper, whose characteristics and method of operation are explained above, and since the properties of this damper is changed, in terms of its ability to absorb vibrations or chocks to which it is exposed and dissipate energy, and that is through varying the voltage value applied to the coils. So for each vibration received by this damper a certain voltage value that achieves the best response. Since this damper is used in the car suspension system, the driver is not able to determine the appropriate voltage value. First, he is driving on a random road, second, it is unable to get off his car and check the surface of the road he will pass. Also, using one value for the voltage in dealing with different types of road surface will cause many problems. For example, instead of being a cause to reduce vibrations transferred to the car body, it may increase them. So, each surface has to be treated with a suitable voltage value. Therefore, in this study, a solution has been found, and if it is not ideal for different road types, it performs a good function by reducing the vibrations transmitted to the passengers.

MATLAB Simulink software is used to create eight DOF full car model with driver seat, and then use the same model and implanted to it a MR damper, to finally have two models, model without MR damper (only passive damper) and model with MR damper, it should be noted that this model has four MR dampers located in the four corners of the vehicle body and implanted between the sprung and unsprung masses. Thus, the response of each model can be obtained and compared for the same input. After applying the road profile, the comparison has been done between these two models with respect to the sprung mass displacement, so the goal was to find the value of the voltage that achieved the least displacement, in term of response peak (by trial and error method). Thus, after changing the...
amplitude of the speed hump and the velocity of the vehicle and after doing all comparisons, a set of voltage values have been obtained that suit each input and achieved the best possible function. As discussed above, it is very difficult or rather impossible for the driver, each time he passes on an uneven road surface, to get off his car and measure the height of the surface and determine the speed that passes over it exactly, also, one voltage value cannot be adopted in dealing with the different road surface, was form it is necessary to find a solution for this problem. ANFIS was adopted as a solution in this study. When the comparison was made, large number of data was obtained, includes the appropriate voltage for each amplitude and velocity. These data have been used to train the ANFIS controller.

This control system has two inputs (speed and road profile) and one output (voltage) as shown in Fig. 7.

![Fig. 7: ANFIS inputs and output](image)

The figure below shows the loaded data into the ANFIS editor GUI.

![Fig. 8: Loaded training data plot in the ANFIS editor](image)

The acquired dynamic data is considered as a group training data for the ANFIS controller. A FIS file is generated in the GUI having nine Gaussian membership functions for each inputs and output. Finally, we train the FIS file with hydride optimization taking zero tolerance error and 10 epochs. The training error comes out to be 0.04841. Fig. 9 shows the surface viewer of the ANFIS model.
The Simulink full car model with MR damper and ANFIS controller is shown in Fig. 10.

Since this study is based on providing ride comfort in the field of transportation, by reducing vibrations transmitted to the vehicle body and then to the driver and passengers. Therefore, taking into consideration any problem that may arise on the control system that was established is necessary in order to achieve the goal of this research and increase the effectiveness of the controller to achieve better performance. The system that was adopted in this study is based on MR dampers, which effectiveness to absorbing vibrations and shocks changes with respect to the voltage value applied to these dampers, in which, for each road disturbance a voltage value that achieve the best performance of the vehicle when overcoming this obstacle. The voltage prediction is done, as indicated above, by an intelligent controller, and that by reading the vehicle speed and the road profile. But in the event of any disruption to the voltage value sent to the dampers, whether it is an increase or decrease in this value, this will reduce the effectiveness of the control system. Therefore, this situation had to be taken into consideration, and find a solution to eliminate any disturbance affecting the desired voltage value. The controller that was adopted to deal with this case is the PID controller, which was implanted to the system as shown in the block diagram below (Fig. 11). The PID controller calculates the error between the actual and the desired voltage, and then attempts to minimize this error and cancelling any value of noise. The noise was inserted to the model in the form of second order transfer function that has a random value.
4. RESULTS AND DISCUSSION

4.1. Without Noise

The following figures (Fig. 12-15) show the response of the body and seat displacements at 0.12 and 0.1 m amplitude for 120 and 60 Km/h speeds, without and with MR damper.

Fig.12: Sprung mass vertical displacements at 0.12 m amplitude for 60 and 120 Km/h speeds

Fig.13: Sprung mass vertical displacements at 0.1 m amplitude for 60 and 120 Km/h speeds
As shown in Fig. 12-15, by comparing the system with MR damper to that without this damper in terms of response peak: For sprung mass vertical displacement, the improvement is 58.72% at 120 Km/h while it is decreased to 40.71% at 60 Km/h in case of 0.12 m amplitude when using the proposed controller. At 0.1 m amplitude the response is improved by 58.26% at 120 Km/h and by 40.73% at 60 Km/h. For driver seat displacement, at 120 Km/h and 0.12 m amplitude the improvement achieved is 59.67% and at 60 Km/h this value decreased to be 24.95%. The response is improved by 59.2% in case of 0.1 m amplitude and 120 Km/h speed while the value of improvements is decreased to 25.37% at 60 Km/h with the same amplitude.
Fig. 16: Sprung mass vertical displacements’ peak for different speeds (a) at 0.12 m amplitude and (b) at 0.1 m amplitude

Fig. 17: Driver seat vertical displacements’ peak for different speeds (a) at 0.12 m amplitude and (b) at 0.1 m amplitude.

The above diagrams show the variation of the response peak for the sprung mass and seat displacement at different speeds. These diagrams show that the improvement of the response, when using MR damper based ANFIS controller compared to passive system is in its highest value (approximate 60%) at high speed while this value decreased to be 0% approximately at low speed.

4.2. In Case of Noise

Fig. 18 below shows the obtained voltage when the vehicle travels over a hump that has a 0.1 m height and at 80 Km/h, in the normal case (in absence of disturbance), also it shows the voltage when a noise affected it, with and without PID controller.
As shown in the above figure, in case of PID, the voltage value varies to reach a maximum value equal to 2.3 V, so it increase about 9.6% compared to the normal value (2.08 V), while its value, in absence of the PID, varies to reach 3.18 V as peak value, which is equal to 134.5 % of the normal value. This causes problems in the system, as mentioned above, in the absence of PID controller.

Fig. 19 shows the response of the system (Driver Seat Displacement), for the same case of Fig. 20 (Amplitude = 0.1 m and speed = 80 Km/h).

![Fig.19: Driver seat displacements](image)

The response of the system without MR damper and that when semi active suspension system is used, in absence of noise, were discussed in the previous section. Fig. 19 shows that when the voltage affected by noise and the PID was not implanted to the system, the seat displacement ‘peak increase 7.73% compared to the response with MR damper in absence of noise. But when PID controller was implanted to the model, the response takes approximately the same path that of the seat displacement in the normal case with MR damper.

5. CONCLUSIONS
A. Providing ride comfort was the objective of this study, in order to achieve this goal, an eight DOF full car model was developed using Matlab Simulink.
B. The model was assumed to travel on a speed hump has amplitude that varies from 0.01 to 0.2 m with different speeds from 20 to 120 Km/h.
C. MR damper was implanted to the model to study its effect on ride comfort.
D. ANFIS controller was used to find the optimum voltage value applied to the damper.
E. Results show that the MR damper improves the vehicle ride comfort over the passive suspension system.
F. The improvement is in the form of, approximately, 60% reduction in the response peak of seat and sprung mass vertical displacements at 120 km/h speed, while its level gradually reduced with low speed.
G. The PID controller plays an important role by cancelling the effect of noise that affects the voltage sending to the MR damper.

REFERENCES


APPENDIX

Table 1: Nomenclature and parameters used in the full car model  
Reference: (Shirahatti, Prasad, Panzade, & Kulkarni, 2008)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_p$</td>
<td>Passenger seat mass</td>
<td>100</td>
<td>Kg</td>
</tr>
<tr>
<td>$M$</td>
<td>Sprung mass</td>
<td>2160</td>
<td>Kg</td>
</tr>
<tr>
<td>$M_1$</td>
<td>Front left side un-sprung mass</td>
<td>85</td>
<td>Kg</td>
</tr>
<tr>
<td>$M_2$</td>
<td>Rear left side un-sprung mass</td>
<td>60</td>
<td>Kg</td>
</tr>
<tr>
<td>$M_3$</td>
<td>Front right side un-sprung mass</td>
<td>85</td>
<td>Kg</td>
</tr>
<tr>
<td>$M_4$</td>
<td>Rear right side un-sprung mass</td>
<td>60</td>
<td>Kg</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Passenger seat stiffness</td>
<td>98935</td>
<td>(N/m)</td>
</tr>
<tr>
<td>$K_1$</td>
<td>Front left side suspension spring stiffness</td>
<td>96861</td>
<td>(N/m)</td>
</tr>
<tr>
<td>$K_2$</td>
<td>Rear left side suspension spring stiffness</td>
<td>52310</td>
<td>(N/m)</td>
</tr>
<tr>
<td>$K_3$</td>
<td>Front right side suspension spring stiffness</td>
<td>96861</td>
<td>(N/m)</td>
</tr>
<tr>
<td>$K_4$</td>
<td>Rear right side suspension spring stiffness</td>
<td>52310</td>
<td>(N/m)</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Tire stiffness</td>
<td>200000</td>
<td>(N/m)</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Passenger seat damping coefficient</td>
<td>615</td>
<td>(Ns/m)</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Front left side suspension damping coefficient</td>
<td>2460</td>
<td>(Ns/m)</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Rear left side suspension damping coefficient</td>
<td>2281</td>
<td>(Ns/m)</td>
</tr>
<tr>
<td>$C_3$</td>
<td>Front right side suspension damping coefficient</td>
<td>2460</td>
<td>(Ns/m)</td>
</tr>
<tr>
<td>$C_4$</td>
<td>Rear right side suspension damping coefficient</td>
<td>2281</td>
<td>(Ns/m)</td>
</tr>
<tr>
<td>$a$</td>
<td>Center of gravity (CG) location from front axle</td>
<td>1.524</td>
<td>(m)</td>
</tr>
<tr>
<td>$b$</td>
<td>Center of gravity (CG) location from rear axle</td>
<td>1.156</td>
<td>(m)</td>
</tr>
<tr>
<td>$2W$</td>
<td>Wheel track</td>
<td>1.45</td>
<td>(m)</td>
</tr>
<tr>
<td>$X_p$</td>
<td>Distance of seat position from CG of sprung mass</td>
<td>0.234</td>
<td>(m)</td>
</tr>
<tr>
<td>$Y_p$</td>
<td>Distance of seat position from CG of sprung mass</td>
<td>0.375</td>
<td>(m)</td>
</tr>
<tr>
<td>$I_x$</td>
<td>Mass moment of inertia for pitch</td>
<td>946</td>
<td>(kg- m$^2$)</td>
</tr>
<tr>
<td>$I_y$</td>
<td>Mass moment of inertia for roll</td>
<td>4140</td>
<td>(kg- m$^2$)</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>Road input at front left side</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_2$</td>
<td>Road input at rear left side</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_3$</td>
<td>Road input at front right side</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_4$</td>
<td>Road input at rear left side</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>